EEG and MEG source modeling: Beamformer Method

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Linear Forward/Inverse Problem

Measured data is connected with model parameters via Gain Matrix

$$\mathbf{d} = \underline{\underline{\mathbf{G}}}\mathbf{m}$$

d: Data vector of size $N \times 1$
m: Model parameter vector of size $M \times 1$
$\underline{\underline{\mathbf{G}}}$: Gain matrix containing forward calculation with size $N \times M$

- In case of $N = M$ there is one unique solution

$$\mathbf{m} = \underline{\underline{\mathbf{G}}}^{-1}\mathbf{d}$$

- For ill-posed problems inversion methods are used to solve equation system
- In non-linear case **d** and **m** are connected via operator $G(\mathbf{m})$

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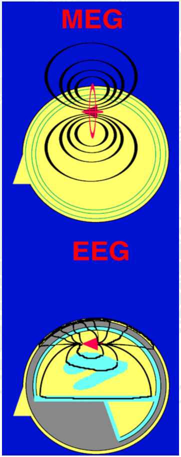
What is measured in MEG and EEG?

- Microskopic current flow is generated by neutral activity
- Electric potential (EEG) and magnetic field (MEG) at the surface is measured
- Fields can be expressed as a superposition of dipoles
- We want information about:
    - Position
    - Strength
    - Direction



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Potential and Magnetic Field generated by current flow

- EEG and MEG are quasi-static

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} d\mathbf{r}'$$

- current density consists of primary current and volume current flow

$$\mathbf{j}(\mathbf{r}') = \mathbf{j}^P(\mathbf{r}') + \mathbf{j}^V(\mathbf{r}') = \mathbf{j}^P(\mathbf{r}') - \sigma(\mathbf{r}')\nabla V(\mathbf{r}')$$

- Forward Problem: Calculate the Potential/Magnetic field at $\mathbf{r}$ generated by current at point $\mathbf{r}'$

$$V_0(\mathbf{r}) = \frac{1}{4\pi\sigma_0} \int \mathbf{j}^P(\mathbf{r}') \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} d\mathbf{r}'$$
$$\mathbf{B}_0(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \mathbf{j}^P(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} d\mathbf{r}'$$

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Current Dipole

$$\mathbf{j}^P(\mathbf{r}') = \mathbf{q}\,\delta(\mathbf{r}' - \mathbf{r})$$

with moment $\mathbf{q} = \int \mathbf{j}^P(\mathbf{r}')d\mathbf{r}'$

- Dipole moment represents neural activity of a source region
- For measurements at the surface different resistivities of brain, skull, scalp has to be considered
- Spherical head model handles this a set of concentric homogeneous shells
- Only radial component away from brain center is measured where volume current terms vanish

$$B_r(\mathbf{r}) = \frac{\mathbf{r}}{r} \cdot \mathbf{B}(\mathbf{r}) = \frac{\mathbf{r}}{r} \cdot \mathbf{B}_0(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{r} \times \mathbf{r}_q}{r|\mathbf{r} - \mathbf{r}_q|^3} \cdot \mathbf{q}$$

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Algebraic Formulation of Forward Model (Baillet et al. 2001)

- Moment $\mathbf{q}$ located at $\mathbf{r}_q$ is seperated into dipole magnitude q and orientation $\Theta = \{\theta, \varphi\}$
- Electric Potential/Magnetic Field generated by dipole:

$$m(\mathbf{r}) = a(\mathbf{r}, \mathbf{r}_q, \Theta)\, q$$

  where $a(\mathbf{r}, \mathbf{r}_q, \Theta)$ is the solution to the magnetic ord electric forward problem with unit amplitude and orientation Θ
- Superposition of multiple dipoles yield

$$m(\mathbf{r}) = \sum_i a(\mathbf{r}, \mathbf{r}_{qi}, \Theta_i)\, q_i$$

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Algebraic Formulation of Forward Model

$$
\begin{aligned}
\mathbf{m} &= \begin{bmatrix} m(\mathbf{r}_1) \\ \vdots \\ m(\mathbf{r}_N) \end{bmatrix} = \begin{bmatrix} a(\mathbf{r}_1, \mathbf{r}_{q1}, \Theta_1) & \cdots & a(\mathbf{r}_1, \mathbf{r}_{qp}, \Theta_p) \\ \vdots & \ddots & \vdots \\ a(\mathbf{r}_N, \mathbf{r}_{q1}, \Theta_1) & \cdots & a(\mathbf{r}_N, \mathbf{r}_{qp}, \Theta_p) \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_N \end{bmatrix} \\
&= \underline{\underline{\mathbf{A}}}\left(\{\mathbf{r}_{qi}, \Theta_i\}\right) \mathbf{S}^T
\end{aligned}
$$

where m are the EEG/MEG measurements at all N electrodes, $\underline{\underline{\mathbf{A}}}$ is the gain matrix relating p dipoles to the N sensor locations and **S** contains the source amplitudes

- Each column of $\underline{\underline{\mathbf{A}}}$ relates one dipole to the sensor array

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Time-dependent Forward Model

$$\underline{\underline{\mathbf{M}}} = \begin{bmatrix} m(\mathbf{r}_1, 1) & \cdots & m(\mathbf{r}_1, T) \\ \vdots & \ddots & \vdots \\ m(\mathbf{r}_N, 1) & \cdots & m(\mathbf{r}_N, T) \end{bmatrix} = \underline{\underline{\mathbf{A}}}\left(\{\mathbf{r}_{qi}, \Theta_i\}\right) \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_p^T \end{bmatrix}$$

for timesteps t=1,...,T

Beamforming as Inversion Method

- Uses spatial filtering to discriminate between signals from region of interest and originating elsewhere
- As the dipole orientation is unknown, we need a set of spatial filters for each cartesian axis denoted as $\{\Theta_1, \Theta_2, \Theta_3\}$
- Output is a 3×1 vector $\mathbf{y} = \underline{\underline{\mathbf{W}}}^T \mathbf{m}$ with a $3 \times N$ spatial filtering matrix $\underline{\underline{\mathbf{W}}}^T$
- Spatial filter should filter out all signals that are not generated by the dipole at location r_q i. e.

$$\underline{\underline{\mathbf{W}}}^T \underline{\underline{\mathbf{A}}}(\mathbf{r}) = \begin{cases} \underline{\underline{\mathbf{I}}} & |\mathbf{r} - \mathbf{r}_q| \leq \delta : \textit{passband} \\ 0 & |\mathbf{r} - \mathbf{r}_q| > \delta : \textit{stopband} \end{cases}$$

with $\underline{\underline{\mathbf{A}}}(\mathbf{r}) = [\mathbf{a}(\mathbf{r}, \Theta_1), \mathbf{a}(\mathbf{r}, \Theta_2), \mathbf{a}(\mathbf{r}, \Theta_3)]$ as the $N \times 3$ forward matrix for 3 orthogonal dipoles at location $\mathbf{r}$

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Spatial Filtering Matrix

- Problem to solve: Find the weighting coefficients for the filter matrix
- Constraints needed to determine $\underline{\underline{\mathbf{W}}}$
- For EEG/MEG Linearly constrained minimum variance (LCMV) method is commonly used

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LCMV Beamforming

- Nulling the interfering signal weights by minimizing the power of the beamformer subject to a unity gain constraint at $\mathbf{r}_q$

$$\min_{W^T} tr\left\{\underline{\underline{\mathbf{C}}}(y)\right\} \text{subject to } \underline{\underline{\mathbf{W}}}^T \underline{\underline{\mathbf{A}}}(\mathbf{r}_q) = \underline{\underline{\mathbf{I}}}$$

where the covariance matrix $\underline{\underline{\mathbf{C}}}(y) = E\left[\mathbf{y}\mathbf{y}^T\right] = \underline{\underline{\mathbf{W}}}^T \underline{\underline{\mathbf{C}}}(m) \underline{\underline{\mathbf{W}}}$ and

$$\underline{\underline{\mathbf{C}}}(m) = E\left[\mathbf{m}\mathbf{m}^T\right]$$

- Solving this with Lagrange multipliers yields (van Veen et al. 1988):

$$\underline{\underline{\mathbf{W}}} = \left[\underline{\underline{\mathbf{A}}}(\mathbf{r}_q)^T \underline{\underline{\mathbf{C}}}(m)^{-1} \underline{\underline{\mathbf{A}}}(\mathbf{r}_q)\right]^{-1} \underline{\underline{\mathbf{A}}}(\mathbf{r}_q)^T \underline{\underline{\mathbf{C}}}(m)^{-1}$$

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Estimate Brain Activity

1. Determine Spatial Filter Matrix $\underline{\underline{\mathbf{W}}}$
2. Apply filter to all snapshot vectors $\mathbf{m}(t)$ for t=1,..,T to estimate dipole moment of a source at $\mathbf{r}_q$
3. Change location $\mathbf{r}_q$ and therefore estimate neural activity in different brain regions

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Beamforming: Pro and Contra Factors

Pro:

- Operate on raw data (no averaging necessary)
- Induced brain activity can be analysed
- Independent from a priori knowledge about number of sources

Contra:

- Sensitive to constraints
- Sensitive to correlation of activity (cancelling out, partial interfering)
- Sensitive to noise, periods without signal (several methods available to treat noise effects)

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Thank you for your attention!

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